AESB 2320 Part 1 Exam 13 March solution

1. (a) The derivation here follows that of the falling film BSL1 and BSLK sect 2.2 up through the momentum balance + integration for τ_{xz} up to the application of the BC. at $x=0$. So

$$\tau_{xz}(x) = \rho g(\cos\beta)x + C_1$$

In this case, $\tau_{xz} = \tau_{xz}^*$ at $x=0$. $C_1 = \tau_{xz}^*$

$$\tau_{xz}(x) = \rho g(\cos\beta)x + \tau_{xz}^*$$

Plugging in Newton's law of viscosity,

$$\tau_{xz} = -\mu\frac{dv_z}{dx} = \rho g(\cos\beta)x + \tau_{xz}^* \qquad \boxed{I}$$

b)
$$\frac{dv_z}{dx} = -\frac{\rho g}{\mu}(\cos\beta)x - \frac{\tau_{xz}^*}{\mu}$$

$$v_z = -\frac{\rho g}{2\mu}(\cos\beta)x^2 - \frac{\tau_{xz}^*}{\mu}x + C_2$$

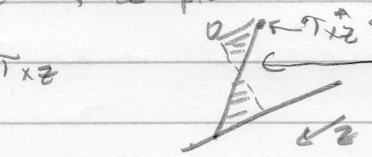
B.C.: $v_z = 0$ at $x=\delta$:

$$0 = -\frac{\rho g}{2\mu}(\cos\beta)\delta^2 - \frac{\tau_{xz}^*}{\mu}\delta + C_2$$

$$C_2 = \frac{\rho g}{2\mu}(\cos\beta)\delta^2 + \frac{\tau_{xz}^*}{\mu}\delta$$

$$v_z = \frac{\rho g\delta^2}{2\mu}(\cos\beta)\left(1-\left(\frac{x}{\delta}\right)^2\right) + \frac{\tau_{xz}^*\delta}{\mu}\left(1-\frac{x}{\delta}\right)$$

c) Since $\tau_{xz}^* < 0$, a plot of $\tau_{xz}(x)$ looks something like this:



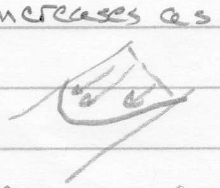
τ_{xz} increases with x according to eq. I above.

From Newton's law of viscosity, $-\mu\frac{dv_z}{dx} = \tau_{xz}$

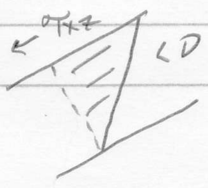
If $\tau_{xz} < 0$, v_z increases w/ x

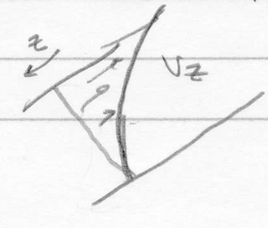
" , " decreases "

According to the picture above, $v_z = 0$ at $x=\delta$; it increases as x decreases near wall, then increases as x decreases to top of film.

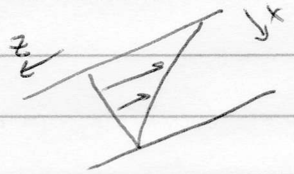


If $\tau_{xz}(x=\delta) \leq 0$, velocity decreases with increasing x throughout film. Ans: $\tau_{xz}^* \leq -\rho g\delta(\cos\beta)$ (see eq. I)





c) An alternate derivation



If $v_z \leq 0$ everywhere, and $v_z = 0$ at $x = \delta$ (B.C.), then $\frac{dv_z}{dx} \leq 0$ at $x = \delta$

from answer to part (a),

$$\frac{dv_z}{dx} = \frac{\rho g \delta^2}{2\mu}(\cos\beta)(-2)\frac{x}{\delta^2} - \frac{\tau_{xz}^{*}\,\delta}{\mu}\left(\frac{1}{\delta}\right)$$

at $x = \delta$

$$\frac{dv_z}{dx} = \frac{-\rho g}{\mu}(\cos\beta)\,\delta - \frac{\tau_{xz}^{*}}{\mu}$$

The smallest magnitude value of τ_{xz}^{*} for which $\frac{dv_z}{dx} \leq 0$ at $x = \delta$ is that for which $\frac{dv_z}{dx} = 0$

$$0 = -\rho g \delta(\cos\beta) - \tau_{xz}^{*}$$

$$\tau_{xz}^{*} = -\rho g \delta(\cos\beta)$$ as shown above

2. a) If flow is laminar then Q is given by Eq. (C) of problem 2.B.4 (BSLK p. 71), which was derived in the homework. In this case, $Q \sim (2B)^3$. The flow rate would be $(2)^3 = 8\times$ as large.

b) For highly turbulent flow, the hydraulic-radius approx. applies. $f = \text{const.} = \frac{1}{4}\left(\frac{D_h}{L}\right)\left(\frac{\Delta p}{\frac{1}{2}\rho v^2}\right)$ (Eq. 6.1-4, BSLK p. 164)

$v \sim \sqrt{D_h} \sim \sqrt{4B}$ ← f = const at large Re: see Fig. 6.2-2)

$Q = v \cdot \text{area} \sim v\,(2B) \sim B^{3/2}$

Flow rate increases like $(2)^{1.5} = 2.83\times$

See additional notes on next p.

3) Fluid in the tank is at rest. There is a sudden constriction, 3.2 m of tubing, 2 sharp 90° elbows,

Eq. 7.5-11:

$$\frac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1) + \frac{p_2 - p_1}{\rho} = \hat{W}_m - \frac{1}{2}v^2 \frac{4L}{D_h} f - \Sigma\left(\frac{v^2}{2} e_v\right)_i$$

What is v? If $Q = 10^{-4}\,m^3$, $v = Q/[\pi R^2] = 10^{-4}/[\pi(0.005)^2] = 1.27$

$h_2 - h_1 = 1.8\,m$

$p_1 = 1\,atm + 0.5\rho g = 1\,atm + (0.5)(1000)(9.8)$

$p_2 = 1\,atm$; $p_2 - p_1 = -0.5(1000)(9.8) = -49\,000$

$Re = Dv\rho/\mu = (0.01)(1.27)(1000)/0.001) = 12700$

$f \simeq 0.0072$

For square elbows, $e_v \approx 1.6$ $\boxed{\text{I}}$ For entrance, $e_v = 0.45$ $\boxed{\text{II}}$

$$\frac{1}{2}((1.27)^2 - 0) + 9.8(1.8) - \frac{4900}{1000} = \hat{W}_m - \frac{1}{2}(1.27)^2 \frac{4(3.2)}{(0.01)}(0.0072)$$
$$- (1.27)^2 \frac{1}{2}((1.6) \times 2 + 0.45)$$

$$0.806 + 17.64 - 4.9 = \hat{W}_m - 7.43 - 2.94$$

$$\hat{W}_m = 23.9$$

The rate of work required is

$$\rho Q \hat{W}_m = 1000\,(10^{-4})\,23.9 = 2.39 \text{ Watts}$$

Footnotes to problem 3

$\boxed{\text{I}}$ actually, between 1.3 and 1.9. Avg value in that range is OK.

$\boxed{\text{II}}$ If rounded, 0.05; if abrupt 0.45. Either OK, but I use 0.45 here.

More on problem 2:

a) Note that the hydraulic-radius approx. does not apply to laminar flow.

b) Some students confused this problem with the homework problem on flow through the crack in the Titanic. In that problem, Q is known (fixed); in this problem $\frac{\Delta P}{L}$ is fixed but Q is not known.

My intended meaning in the problem statement was that the tube was rough, but <u>for</u> <u>simplicity</u> we would assume (k/D_h) does not change as $D_h = 4B$ changes. Some students interpreted this as roughness is zero. In that case, <u>only</u> in the range $2100 \lesssim Re \lesssim 10^5$,

(Fig 6.2-2): $f \sim \frac{0.0791}{Re^{1/4}} \sim \frac{\Delta P \, D_h}{V^2}$ (Eq. 6.1-4)

$$\text{Thus } V^2 \propto Re^{1/4} \propto \left(\frac{D_h V \rho}{\mu}\right)^{1/4} D_h$$

$$V^{1.75} \propto D_h^{1.25} \qquad (D_h = 4B)$$

$$V \propto D_h^{5/7} \; ; \; Q = V \cdot A \propto B^{(5/4)} B = B^{9/4} \text{ rather than } B^{1.5}$$

Also, some students confused this problem with the homework problem involving highly turbulent flow in a tube. In that problem, as here, $V \sim D^{1/2}$, but area $\sim D^2$. Here area $\sim B$, not B^2.